

UNIT -1

Fundamentals

Symbol – An atomic unit, such as a digit, character, lower-case letter, etc. Sometimes a word. [Formal language does not deal with the “meaning” of the symbols.]

Alphabet – A finite set of symbols, usually denoted by Σ .

$$\Sigma = \{0, 1\}$$

$$\Sigma = \{0, a, 9, 4\}$$

$$\Sigma = \{a, b, c, d\}$$

String – A finite length sequence of symbols, presumably from some alphabet. $w=0110$

$$y=0aa$$

$$x=aabcaa$$

$$z = 111$$

Special string: ϵ (also denoted by λ)

Concatenation: $wz = 0110111$

Length: $|w| = 4$ $|\epsilon| = 0$ $|x| = 6$

Reversal: $y^R = aa0$

Some special sets of strings:

$$\Sigma^* \quad \text{All strings of symbols from } \Sigma$$

$$\Sigma^+ \quad \Sigma^* - \{\epsilon\}$$

Example: $\Sigma = \{0,1\}$

$$\Sigma^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, 001, \dots\}$$

$$\Sigma^+ = \{0, 1, 00, 01, 10, 11, 000, 001, \dots\}$$

A language is:

A set of strings from some alphabet (finite or infinite). In other words,

Any subset L of Σ^*

Some special languages:

{} The empty set/language, containing no string.

{ ϵ } A language containing one string, the empty string.

Examples:

$$\Sigma = \{0,1\}$$

$$L = \{x \mid x \text{ is in } \Sigma^* \text{ and } x \text{ contains an even number of } 0^{\text{'s}}\}$$

$$\Sigma = \{0, 1, 2, \dots, 9, .\}$$

$$L = \{x \mid x \text{ is in } \Sigma^* \text{ and } x \text{ forms a finite length real number}\}$$

$$= \{0, 1.5, 9.326, \dots\}$$

$$\Sigma = \{a, b, c, \dots, z, A, B, \dots, Z\}$$

$$L = \{x \mid x \text{ is in } \Sigma^* \text{ and } x \text{ is a Pascal reserved word}\}$$

$$= \{\text{BEGIN, END, IF, } \dots\}$$

$$\Sigma = \{\text{Pascal reserved words}\} \cup \{ (,), ., :, ;, \dots\} \cup \{\text{Legal Pascal identifiers}\} \\ L = \{x \mid x \text{ is in } \Sigma^* \text{ and } x \text{ is a syntactically correct Pascal program}\}$$

$$\Sigma = \{\text{English words}\}$$

$$L = \{x \mid x \text{ is in } \Sigma^* \text{ and } x \text{ is a syntactically correct English sentence}\}$$

Regular Expression

- A regular expression is used to specify a language, and it does so precisely.
- Regular expressions are very intuitive.
- Regular expressions are very useful in a variety of contexts.
- Given a regular expression, an NFA- ϵ can be constructed from it automatically.
- Thus, so can an NFA, a DFA, and a corresponding program, all automatically!

Definition:

Let Σ be an alphabet. The regular expressions over Σ are:

\emptyset Represents the empty set $\{\}$

ϵ Represents the set $\{\epsilon\}$

a Represents the set $\{a\}$, for any symbol a in Σ

Let r and s be regular expressions that represent the sets R and S , respectively.

$r+s$ Represents the set $R \cup S$ (precedence 3)

rs Represents the set RS (precedence 2)

r^* Represents the set R^* (highest precedence)

(r) Represents the set R (not an op, provides precedence)

If r is a regular expression, then $L(r)$ is used to denote the corresponding language.

Examples:

Let $\Sigma = \{0,1\}$

$(0+1)^*a0$ All strings of 0's and 1's, beginning with a 0

$(0+1)^*1$ All strings of 0's and 1's, ending with a 1

$(0+1)^*0(0+1)^*$ All strings of 0's and 1's containing at least one 0
 $(0+1)^*0(0+1)^*0(0+1)^*$ All strings of 0's and 1's containing at least two

0's
 $(0+1)^*01^*01^*$ All strings of 0's and 1's containing at least two

0's
 $(101^*0)^*$ All strings of 0's and 1's containing an even number of 0's

$1^*(01^*01^*)^*$ All strings of 0's and 1's containing an even number of 0's

$(1^*01^*0)^*1^*$ All strings of 0's and 1's containing an even number of 0's

Identities:

1. $\emptyset u = u\emptyset = \emptyset$ Multiply by 0

2. $\epsilon u = u\epsilon = u$ Multiply by 1

3. $\emptyset^* = \epsilon$

4. $\epsilon^* = \epsilon$

5. $u+v = v+u$

$$6. u + \emptyset = u$$

$$7. u + u = u$$

$$8. u^* = (u^*)^*$$

$$9. u(v+w) = uv+uw$$

$$10. (u+v)w = uw+vw$$

$$11. (uv)^*u = u(vu)^*$$

$$12. (u+v)^* = (u^*+v)^*$$

$$=u^*(u+v)^*$$

$$=(u+vu^*)^*$$

$$= (u^*v^*)^*$$

$$=u^*(vu^*)^*$$

$$=(u^*v)^*u^*$$

Finite State Machines

A finite state machine has a set of states and two functions called the next-state function and the output function

The set of states correspond to all the possible combinations of the internal storage

If there are n bits of storage, there are 2^n possible states

The next state function is a combinational logic function that given the inputs and the current state, determines the next state of the system

The output function produces a set of outputs from the current state and the inputs

- There are two types of finite state machines
- In a Moore machine, the output only depends on the current state
- While in a Mealy machine, the output depends both the current state and the current input
- We are only going to deal with the Moore machine.
- These two types are equivalent in capabilities

A Finite State Machine consists of:

K states: $S = \{s_1, s_2, \dots, s_k\}$, s_1 is initial state
N inputs: $I = \{i_1, i_2, \dots, i_n\}$

M outputs: $O = \{o_1, o_2, \dots, o_m\}$

Next-state function $T(S, I)$ mapping each current state and input to next state

Output Function $P(S)$ specifies output

Finite Automata

- Two types – both describe what are called regular languages
 - Deterministic (DFA) – There is a fixed number of states and we can only be in one state at a time

- Nondeterministic (NFA) –There is a fixed number of states but wcan bein multiple states at onetime
- While NFA"s are more expressive than DFA"s, we will see that addingnondeterminism does not let us define any language that cannot be defined by aDFA.
- One way to think of this is we might write a program using a NFA, but then when it is "compiled" we turn the NFA into an equivalentDFA.

Formal Definition of a Finite Automaton

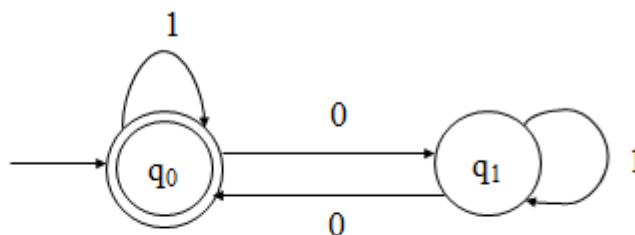
- Finite set of states, typically Q .
- Alphabet of input symbols, typically Σ
- One state is the start/initial state, typically $q_0 // q_0 \in Q$
- Zero or more final/accepting states; the set is typically F . // $F \subseteq Q$
- A transition function, typically δ . Thisfunction
- Takes a state and input symbol asarguments.

Deterministic Finite Automata (DFA)

- A DFA is a five-tuple: $M = (Q, \Sigma, \delta, q_0, F)$
 Q =A finite set ofstates
 Σ =A finite inputalphabet
 q_0 =The initial/starting state, q_0 is in Q
 F =A set of final/accepting states, which is a subset of Q
 Δ =A transition function, which is a total function from $Q \times \Sigma$ to Q
 $\delta: (Q \times \Sigma) \rightarrow Q$ δ is defined for any q in Q and s in Σ , and $\delta(q,s)=q''$ is equal to another state q'' in Q .
 Intuitively, $\delta(q,s)$ is the state entered by M after reading symbol s while in state q .

- For Example #1:

$Q = \{q_0, q_1\}$
 $\Sigma = \{0, 1\}$
 Start state is q_0
 $F = \{q_0\}$



δ :

	0	1
q_0	q_1	q_0
q_1	q_0	q_1

- Let $M=(Q,\Sigma,\delta,q,F)$ be a DFA and let w be in Σ^* . Then w is accepted by M iff

$$\delta(q_0, w) = p \text{ for some state } p \text{ in } F.$$

- Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA. Then the *language accepted* by M is the set:

$$L(M) = \{w \mid w \text{ is in } \Sigma^* \text{ and } \delta(q_0, w) \text{ is in } F\}$$

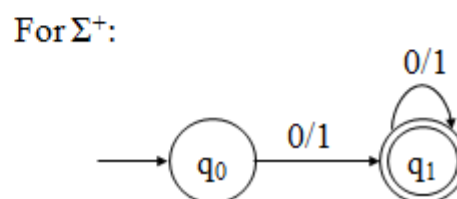
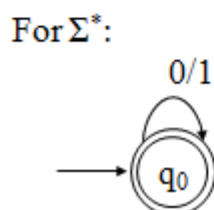
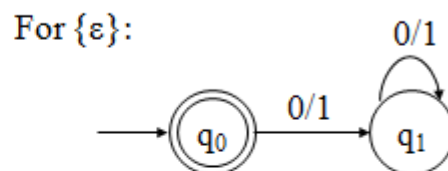
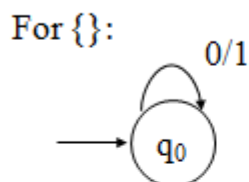
- Another equivalent definition:

$$L(M) = \{w \mid w \text{ is in } \Sigma^* \text{ and } w \text{ is accepted by } M\}$$

- Let L be a language. Then L is a *regular language* iff there exists a DFA M such that $L = L(M)$.

Notes:

- A DFA $M = (Q, \Sigma, \delta, q_0, F)$ partitions the set Σ^* into two sets: $L(M)$ and $\Sigma^* - L(M)$.
- If $L = L(M)$ then L is a subset of $L(M)$ and $L(M)$ is a subset of L .
- Similarly, if $L(M_1) = L(M_2)$ then $L(M_1)$ is a subset of $L(M_2)$ and $L(M_2)$ is a subset of $L(M_1)$.
- Some languages are regular, others are not. For example, if $L_1 = \{x \mid x \text{ is a string of 0's and 1's containing an even number of 1's}\}$ and $L_2 = \{x \mid x = 0^n 1^n \text{ for some } n \geq 0\}$ then L_1 is regular but L_2 is not.
- Let $\Sigma = \{0, 1\}$. Give DFAs for $\{\}$, $\{\epsilon\}$, Σ^* , and Σ^+ .



Nondeterministic Finite Automata (NFA)

An NFA is a five-tuple: $M = (Q, \Sigma, \delta, q_0, F)$

- Q A finite set of states
- Σ A finite input alphabet
- q_0 The initial/starting state, q_0 is in Q
- F A set of final/accepting states, which is a subset of Q
- δ A transition function, which is a total function from $Q \times \Sigma$ to 2^Q

$\delta: (Q \times \Sigma) \rightarrow 2^Q$ 2^Q is the power set of Q, the set of all subsets of Q $\delta(q,s)$ -The set of all states p such that there is a transition

labeled s from q to p $\delta(q,s)$ is a function from $Q \times \Sigma$ to 2^Q (but not to Q)

Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA and let w be in Σ^* . Then w is *accepted* by M iff $\delta(\{q_0\}, w)$ contains at least one state in F.

Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Then the *language accepted* by M is the set: $L(M) = \{w \mid w \text{ is in } \Sigma^* \text{ and } \delta(\{q_0\}, w) \text{ contains at least one state in } F\}$

Another equivalent definition:

$L(M) = \{w \mid w \text{ is in } \Sigma^* \text{ and } w \text{ is accepted by } M\}$

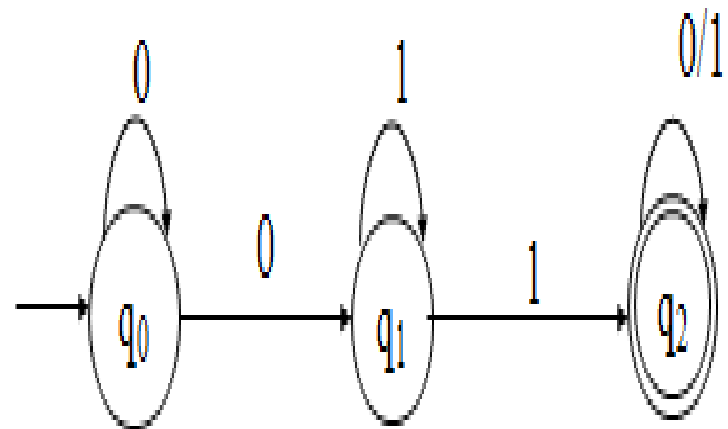
- Example: some 0's followed by some 1's

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{0, 1\}$$

Start state is q_0

$$F = \{q_2\}$$



δ :

	0	1
q_0	$\{q_0, q_1\}$	$\{\}$
q_1	$\{\}$	$\{q_1, q_2\}$
q_2	$\{q_2\}$	$\{q_2\}$

$=\{\epsilon, 0, 00, 1, 11, 111, 01, 10, \dots\}$

$=\{\epsilon, \text{any combination of 0's, any combination of 1's, any combination of 0 and 1}\}$

Hence, L. H. S. = R. H. S. is proved.

3.4 RELATIONSHIP BETWEEN FA AND RE

There is a close relationship between a finite automata and the regular expression we can show this relation in below figure.

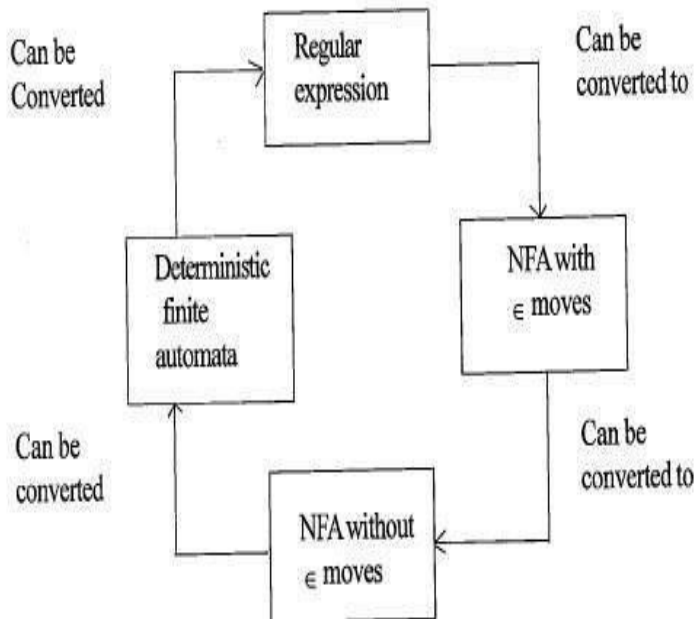


FIGURE : Relationship between FA and regular expression

The above figure shows that it is convenient to convert the regular expression to NFA with ϵ moves. Let us see the theorem based on this conversion.

3.5 CONSTRUCTING FA FOR A GIVEN REs

THEOREM : If r be a regular expression then there exists a NFA with ϵ - moves, which accepts $L(r)$.

Proof : First we will discuss the construction of NFA M with ϵ - moves for regular expression r and then we prove that $L(M) = L(r)$.

Let r be the regular expression over the alphabet Σ .

Construction of NFA with ϵ - moves

Case 1 :

- (i) $r = \phi$

NFA $M = (\{s, f\}, \{\}, \delta, s, \{f\})$ as shown in Figure 1 (a)

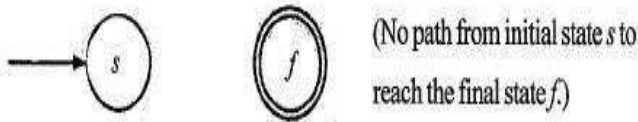


Figure 1 (a)

(ii) $r = \epsilon$

NFA $M = (\{s\}, \{\}, \delta, s, \{s\})$ as shown in Figure 1 (b)

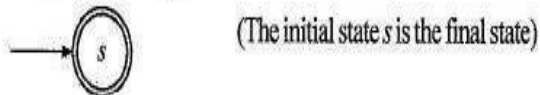


Figure 1 (b)

(iii) $r = a$, for all $a \in \Sigma$,

NFA $M = (\{s, f\}, \Sigma, \delta, s, \{f\})$

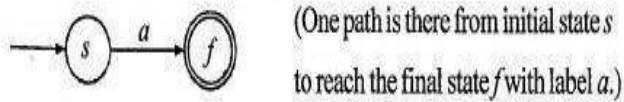


Figure 1 (c)

Case 2: $|r| \geq 1$

Let r_1 and r_2 be the two regular expressions over Σ_1, Σ_2 and N_1 and N_2 are two NFA for r_1 and r_2 respectively as shown in Figure 2 (a).

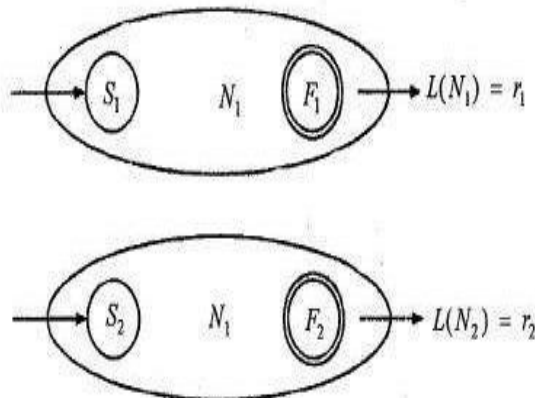
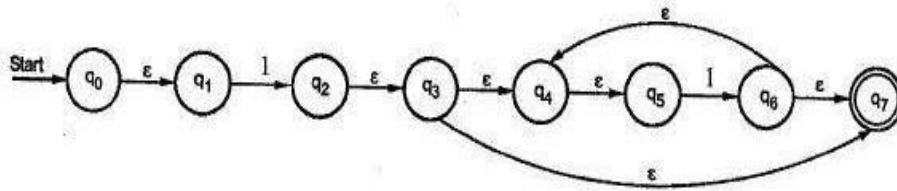


Figure 2 (a) NFA for regular expression r_1 and r_2

The final NFA is



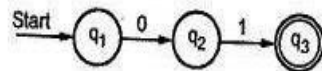
Example 4 : Construct NFA for the r. e. $(01 + 2^*)0$.

Solution : Let us design NFA for the regular expression by dividing the expression into smaller units

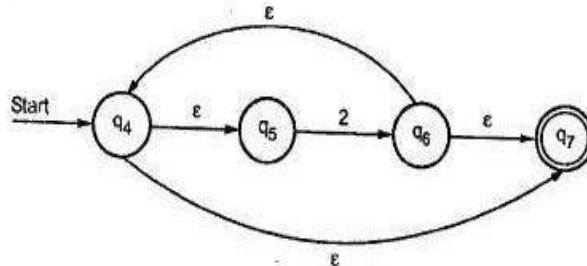
$$r = (r_1 + r_2)r_3$$

where $r_1 = 01$, $r_2 = 2^*$ and $r_3 = 0$

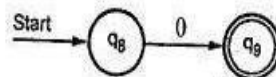
The NFA for r_1 will be



The NFA for r_2 will be



The NFA for r_3 will be



Conversion from NFA to DFA

Suppose there is an NFA $N = \langle Q, \Sigma, q_0, \delta, F \rangle$ which recognizes a language L . Then the DFA $D = \langle Q'', \Sigma, q_0'', \delta'', F'' \rangle$ can be constructed for language L as:

Step 1: Initially $Q'' = \phi$.

Step 2: Add q_0 to Q'' .

Step 3: For each state in Q'' , find the possible set of states for each input symbol using transition function of NFA. If this set of states is not in Q'' , add it to Q'' .

Step 4: Final state of DFA will be all states with contain F (final states of NFA)

Example

Consider the following NFA shown in Figure 1.

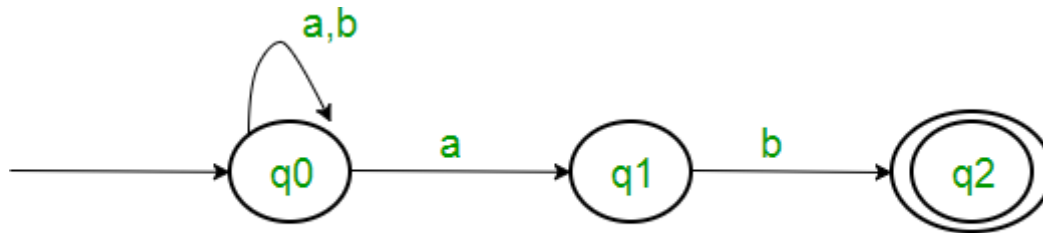


Figure 1

Following are the various parameters for NFA.

$Q = \{ q_0, q_1, q_2 \}$

$\Sigma = (a, b)$

$F = \{ q_2 \}$

δ (Transition Function of NFA)

State	a	b
q0	q0,q1	q0
q1		q2
q2		

Step 1: $Q'' = \phi$

Step 2: $Q'' = \{ q_0 \}$

Step 3: For each state in Q'' , find the states for each input symbol.

Currently, state in Q'' is q_0 , find moves from q_0 on input symbol a and b using transition function of NFA and update the transition table of DFA

δ'' (Transition Function of DFA)

State	a	b
q0	{q0,q1}	q0

Now $\{ q_0, q_1 \}$ will be considered as a single state. As its entry is not in Q'' , add it to Q'' .

So $Q'' = \{ q_0, \{ q_0, q_1 \} \}$

Now, moves from state $\{ q_0, q_1 \}$ on different input symbols are not present in transition table of DFA, we will calculate it like:

$\delta'' (\{ q_0, q_1 \}, a) = \delta (q_0, a) \cup \delta (q_1, a) = \{ q_0, q_1 \}$

$\delta'' (\{ q_0, q_1 \}, b) = \delta (q_0, b) \cup \delta (q_1, b) = \{ q_0, q_2 \}$

Now we will update the transition table of DFA.

δ'' (Transition Function of DFA)

State	a	B
q0	{q0,q1}	q0
{q0,q1}	{q0,q1}	{q0,q2}

Now { q0, q2 } will be considered as a single state. As its entry is not in Q'' , add it to Q'' .

So $Q'' = \{ q0, \{ q0, q1 \}, \{ q0, q2 \} \}$

Now, moves from state {q0, q2} on different input symbols are not present in transition table of DFA, we will calculate it like:

$\delta''(\{q0, q2\}, a) = \delta(q0, a) \cup \delta(q2, a) = \{q0, q1\}$

$\delta''(\{q0, q2\}, b) = \delta(q0, b) \cup \delta(q2, b) = \{q0\}$

Now we will update the transition table of DFA.

δ'' (Transition Function of DFA)

State	a	B
q0	{q0,q1}	q0
{q0,q1}	{q0,q1}	{q0,q2}
{q0,q2}	{q0,q1}	q0

As there is no new state generated, we are done with the conversion. Final state of DFA will be state which has q2 as its component i.e., { q0, q2 }

Following are the various parameters for DFA.

$Q'' = \{ q0, \{ q0, q1 \}, \{ q0, q2 \} \}$

$\Sigma = (a, b)$

$F = \{ \{ q0, q2 \} \}$ and transition function δ'' as shown above. The final DFA for above NFA has been shown in Figure 2.

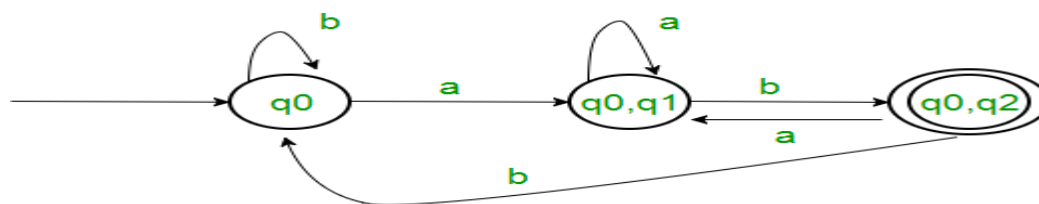


Figure 2

Note : Sometimes, it is not easy to convert regular expression to DFA. First you can convert regular expression to NFA and then NFA to DFA

Application of Finite state machine and regular expression in Lexical analysis: Lexical analysis is the process of reading the source text of a program and converting that source code into a sequence of tokens. The approach of design a finite state machine by using regular expression is so useful to generates token form a given source text program. Since the lexical structure of more or less every programming language can be specified by a regular language, a common way to implement a lexical analysis is to; 1. Specify regular expressions for all of the kinds of tokens in the language. The disjunction of all of the regular expressions thus describes any possible token in the language. 2. Convert the overall regular expression specifying all possible tokens into a deterministic finite automaton (DFA). 3. Translate the DFA into a program that simulates the DFA. This program is the lexical analyzer. To recognize identifiers, numerals, operators, etc., implement a DFA in code. State is an integer variable, δ is a switch statement Upon recognizing a lexeme returns its lexeme, lexical class and restart DFA with next character in source code.

CONTEXT FREE-GRAMMAR

Definition: Context-Free Grammar (CFG) has 4-tuple: $G = (V, T, P, S)$

Where,

V -A finite set of variables or *non-terminals*

T -A finite set of *terminals* (**V and T do not intersect**)

P -A finite set of *productions*, each of the form $A \rightarrow \alpha$,
Where A is in V and α is in $(V \cup T)^*$

Note: that α may be ϵ .

S -A starting non-terminal (S is in V)

Example :CFG:

$G = (\{S\}, \{0, 1\}, P, S)$ P:

$S \rightarrow 0S1$ or just simply $S \rightarrow 0S1 \mid \epsilon$

$S \rightarrow \epsilon$

Example Derivations:

S	=> 0S1	(1)
S	=> ϵ	(2)
	=> 01	(2)
S	=> 0S1	(1)
	=> 00S11	(1)
	=> 000S111	(1)
	=> 000111	(2)

- Note that G “generates” the language $\{0^k 1^k \mid k \geq 0\}$

Derivation (or Parse) Tree

- **Definition:** Let $G = (V, T, P, S)$ be a CFG. A tree is a derivation (or parse) tree if:
 - Every vertex has a label from $V \cup T \cup \{\epsilon\}$
 - The label of the root is S
 - If a vertex with label A has children with labels X_1, X_2, \dots, X_n , from left to right, then

synchronizing set. The Usage of FOLLOW and FIRST symbols as synchronizing tokens works reasonably well when expressions are parsed.

For the constructed table., fill with **synch** for rest of the input symbols of FOLLOW set and then fill the rest of the columns with **error** term.

$$A \rightarrow X_1, X_2, \dots, X_n$$

Terminals

must be a production in P

The first L stands for “Left-to-right scan of input”. The second L stands for “Left-most derivation”. The „1”

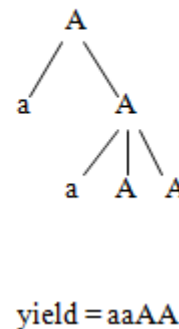
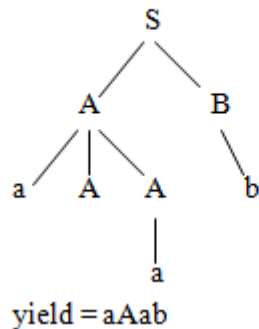
stands for “1 token of look ahead”.

No LL (1) grammar can be ambiguous or left recursive.

LL (1) Grammar:

- If a vertex has label ϵ , then that vertex is a leaf and the only child of its “parent
- More Generally, a derivation tree can be defined with any non-terminal as the root.
- **Example:**

$S \rightarrow AB$
 $A \rightarrow aAA$
 $A \rightarrow aA$
 $A \rightarrow a$
 $B \rightarrow bB$
 $B \rightarrow b$



Notes:

- Root can be any non-terminal
- Leaf nodes can be terminals or non-terminals

If there were no multiple entries in the Recursive decent parser table, the given grammar is LL (1).

If the grammar G is ambiguous, left recursive then the recursive decent table will have at least one multiply defined entry.

The weakness of LL(1) (Top-down, predictive) parsing is that, must predict which production to use.

Error Recovery in Predictive Parser:

Error recovery is based on the idea of skipping symbols on the input until a token in a selected set of synchronizing tokens appear. Its effectiveness depends on the choice of

- A derivation tree with root S shows the productions used to obtain a sentential form.

LL(k)

LL(k) grammar performs a top-down, leftmost parse after reading the string from left-to-right. Here, k is the number of look-aheads allowed.

With the knowledge of k look-aheads, we calculate $FIRST_k$ and $FOLLOW_k$ where:

If the parser looks up entry in the table as synch, then the non terminal on top of the stack is popped in an attempt to resume parsing. If the token on top of the stack does not match the input symbol, then pop the token from the stack.

The moves of a parser and error recovery on the erroneous input id^*+id is as follows:

- $FIRST_k$: k terminals that can be at the beginning of a derived non-terminal
- $FOLLOW_k$: k terminals that can come *after* a derived non-terminal

The basic idea is to create a lookup table using this information from which the parser can then simply go and check what derivation is to be made given a certain input token.

Now, the following text from [here](#) explains strong LL(k):

In the general case, the LL(k) grammars are quite difficult to parse directly. This is due to the fact that the left context of the parse must be remembered somehow.

Each parsing decision is based both on what is to come as well as on what has

already been seen of the input.

The class of LL(1) grammars are so easily parsed because it is strong. The strong LL(k) grammars are a subset of the LL(k) grammars that can be parsed *without* knowledge of the left-context of the parse. That is, each parsing decision is based only on the next k tokens of the input for the current nonterminal that is being expanded.

Formally,

A grammar $(G=N,T,P,S)$ is strong if for any two distinct A-productions in the grammar:

$$A \rightarrow \alpha A \rightarrow \alpha$$
$$A \rightarrow \beta A \rightarrow \beta$$
$$FIRST_k(\alpha FOLLOW_k(A)) \cap FIRST_k(\beta FOLLOW_k(A)) = \emptyset$$

That looks complicated so we'll see it another way. Let's take a textbook example to understand, instead, when is some grammar "weak" or when exactly would we need to know the left-context of the parse.

$$S \rightarrow aAa \quad S \rightarrow aAa$$
$$S \rightarrow bAba \quad S \rightarrow bAba$$
$$A \rightarrow bA \quad A \rightarrow b$$
$$A \rightarrow \epsilon \quad A \rightarrow \epsilon$$

Here, you'll notice that for an LL(2) instance, baba could result from either of the S productions. So the parser needs some left-context to decide whether baba is produced by $S \rightarrow aAa$ or $S \rightarrow bAba$.

Such a grammar is therefore "weak" as opposed to being a strong LL(k) grammar.

Unit-II

BOTTOM UP PARSING:

Bottom-up parser builds a derivation by working from the input sentence back towards the start symbol S. Right most derivation in reverse order is done in bottom-up parsing.

(The point of parsing is to construct a derivation. A derivation consists of a series of rewrite steps)

$$S \Rightarrow r_0 \Rightarrow r_1 \Rightarrow r_2 \Rightarrow \dots \Rightarrow r_n$$



$r_1 \Rightarrow r_n \Rightarrow$ sentence Bottom-up

Assuming the production $A \rightarrow \beta$, to reduce r_i r_{i-1} match some RHS β against r_i then replace β with its corresponding LHS, A. In terms of the parse tree, this is working from leaves to root.

Example - 1:

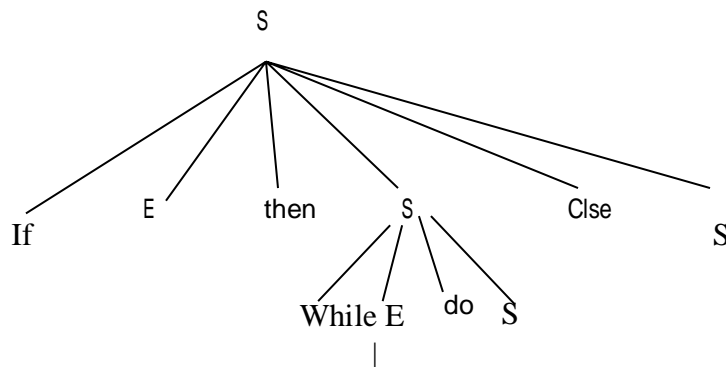
$S \rightarrow$ if E then S else S/while E do S/

print E \rightarrow true/ False/id

Input: if id then while true do print else print.

Parse tree:

Basic idea: Given input string a, "reduce" it to the goal (start) symbol, by looking for substring that match production RHS.



true

\Rightarrow **if E then S else S**

Im

\Rightarrow **if id then S else S**

Im

\Rightarrow **if id then while E do S else S**

Im

\Rightarrow **if id then while true do S else S**

Im

\Rightarrow **if id then while true do print else S**

Im

\Rightarrow if id then while true do print elseprint
 lm
 \Leftarrow if E then while true do print elseprint
 rm
 \Leftarrow if E then while E do print elseprint
 rm
 \Leftarrow if E then while E do S elseprint
 rm
 \Leftarrow if E then S elseprint
 rm
 \Leftarrow if E then S elseS
 rm
 \Leftarrow S
 rm

HANDLE PRUNING:

Keep removing handles, replacing them with corresponding LHS of production, until we reach S.

Example:

$E \rightarrow E+E/E * E/(E)/id$

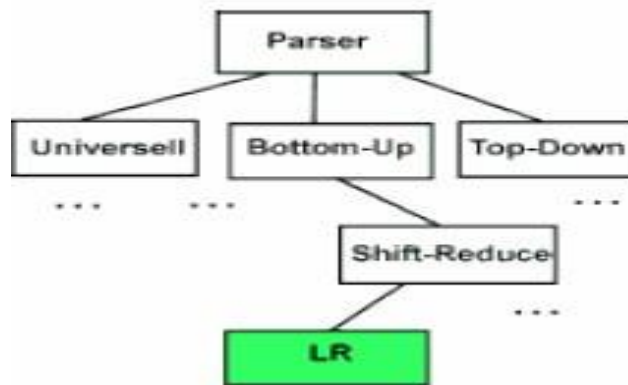
Right-sentential form	Handle	Reducing production
$a+b*c$	A	$E \rightarrow id$
$E+b*c$	B	$E \rightarrow id$

$E+E*C$	C	$E \rightarrow id$
$E+E * E$	$E * E$	$E \rightarrow E * E$
$E+E$	$E+E$	$E \rightarrow E+E$
E		

The grammar is ambiguous, so there are actually two handles at next-to-last step. We can use parser-generators that compute the handles for us

LR PARSING INTRODUCTION:

The "L" is for left-to-right scanning of the input and the "R" is for constructing a rightmost derivation in reverse.



WHY LR-PARSING:

1. LR parsers can be constructed to recognize virtually all programming-language constructs for which context-free grammars can be written.
2. The LR parsing method is the most general non-backtracking shift-reduce parsing method known, yet it can be implemented as efficiently as other shift-reduce methods.
3. The class of grammars that can be parsed using LR methods is a proper subset of the class of grammars that can be parsed with predictive parsers.
4. An LR parser can detect a syntactic error as soon as it is possible to do so on a left-to-right scan of the input.

The disadvantage is that it takes too much work to construct an LR parser by hand for a typical programming-language grammar. But there are lots of LR parser generators available to make this task easy.

LR-PARSERS:

LR(k) parsers are most general non-backtracking shift-reduce parsers. Two cases of interest are $k=0$ and $k=1$. LR(1) is of practical relevance.

„L“ stands for “Left-to-right” scan of input.

„R“ stands for “Rightmost derivation (in reverse)”.

K stands for number of input symbols of look-a-head that are used in making parsing decisions. When (K) is omitted, „K“ is assumed to be 1.

LR(1) parsers are table-driven, shift-reduce parsers that use a limited right context (1 token) for handle recognition.

LR(1) parsers recognize languages that have an LR(1) grammar.

A grammar is LR(1) if, given a right-most derivation

$S \Rightarrow r_0 \Rightarrow r_1 \Rightarrow r_2 \dots \Rightarrow r_{n-1} \Rightarrow r_n \Rightarrow \text{sentence}$.

We can isolate the handle of each right-sentential form r_i and determine the production by which to reduce, by scanning r_i from left-to-right, going atmost 1 symbol beyond the right end of the handle of r_i .

Parser accepts input when stack contains only the start symbol and no remaining input symbol are left.

LR(0) item: (no lookahead)

Grammar rule combined with a dot that indicates a position in its RHS.

Ex-1: $S^1 \rightarrow .S\$$

$S \rightarrow .$

$x S \rightarrow .(L)$

Ex-2: $A \rightarrow XYZ$ generates 4 LR(0) items

$A \rightarrow .XYZ$

$A \rightarrow X.$

$YZ A \rightarrow XY.$

$Z A \rightarrow XYZ.$

$A \rightarrow XY.Z$ indicates that the parser has seen a string derived from XY and is looking for one derivable from Z.

→ LR(0) items play a key role in the SLR(1) table construction algorithm.

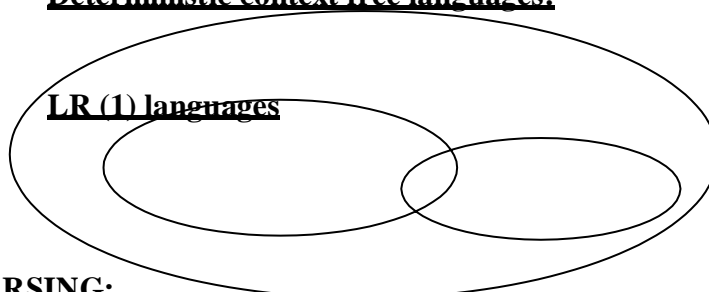
→ LR(1) items play a key role in the LR(1) and LALR(1) table construction algorithms. LR parsers have more information available than LL parsers when choosing a production:

* LR knows everything derived from RHS plus, 'K' lookahead symbols.

* LL just knows, 'K' lookahead symbols into what's derived from RHS.

* **Deterministic context free languages:**

*
*
*
*
*
*
*



LALR PARSING:

Example:

Construct $C = \{I_0, I_1, \dots, I_n\}$ The collection of sets of LR(1) items

For each core present among the set of LR(1) items, find all sets having that core, and replace these sets by their Union# (clus them into a single term)

$I_0 \rightarrow$ same as previous

$I_1 \rightarrow$ “

$I_2 \rightarrow$ “

I_{36} – Clubbing item I_3 and I_6 into one I_{36} item.

$C \rightarrow cC, c/d/\$$

$C \rightarrow cC, c/d/\$$

$C \rightarrow d, c/d/\$$

$I_5 \rightarrow$ same as previous

$I_{47} \rightarrow C \rightarrow d, c/d/\$$

$I_{89} \rightarrow C \rightarrow cC, c/d/\$$

LALR Parsing table construction:

State	Action			Goto	
	c	d	\$	S	C
I_0	S_{36}	S_{47}		1	2
1			Accept		
2	S_{36}	S_{47}			5
36	S_{36}	S_{47}			89
47	r_3	r_3			
5			r_1		
89	r_2	r_2	r_2		

Ambiguous grammar:

A CFG is said to be ambiguous if there exists more than one derivation tree for the given input string i.e., more than one **LeftMost Derivation Tree (LMDT)** or **RightMost Derivation Tree (RMDT)**.

Definition: $G = (V, T, P, S)$ is a CFG is said to be ambiguous if and only if there exist a string in T^* that has more than one parse tree.

where V is a finite set of variables.

T is a finite set of terminals.

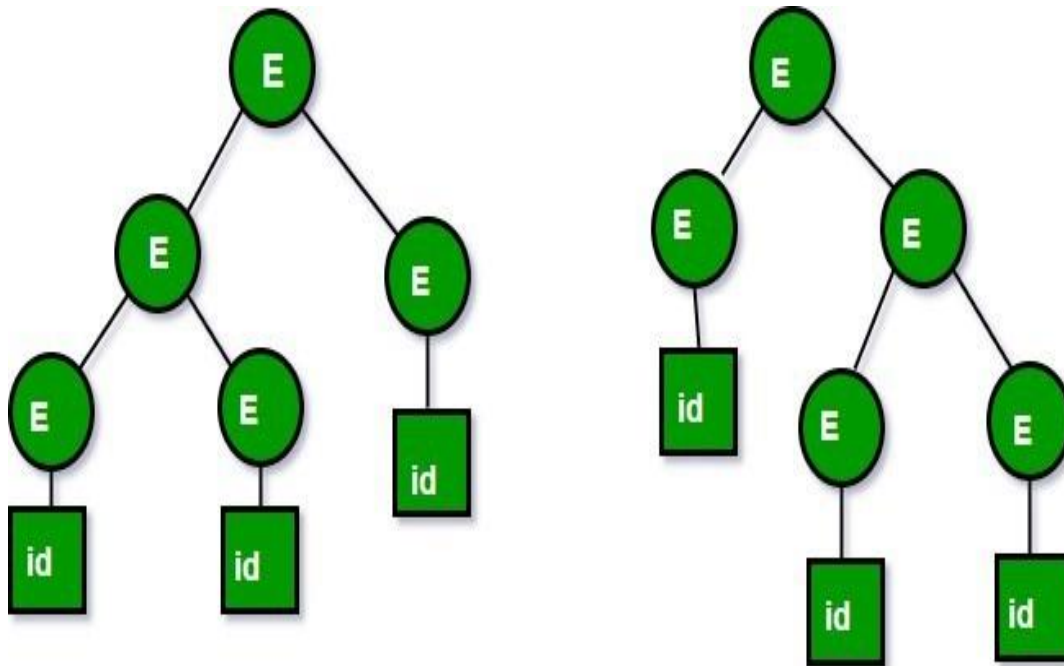
P is a finite set of productions of the form, $A \rightarrow \alpha$, where A is a variable and $\alpha \in (V \cup T)^*$ S is a designated variable called the start symbol.

For Example:

1. Let us consider this grammar : $E \rightarrow E+E|id$

We can create 2 parse tree from this grammar to obtain a string **id+id+id** :

The following are the 2 parse trees generated by left most derivation:



Both the above parse trees are derived from same grammar rules but both parse trees are different. Hence the grammar is ambiguous.

YACC PROGRAMMING

A parser generator is a program that takes as input a specification of a syntax, and produces as output a procedure for recognizing that language. Historically, they are also called compiler-compilers.

YACC (yet another compiler-compiler) is an **LALR(1)** (LookAhead, Left-to-right, Rightmost derivation producer with 1 lookahead token) parser generator. YACC was originally designed for being complemented by Lex.

Input File:

YACC input file is divided in three parts.

```
/* definitions */
....

%%

/* rules */
....

%%

/* auxiliary routines */
....
```

Input File: Definition Part:

- The definition part includes information about the tokens used in the syntax definition:
- %token NUMBER

`%token ID`

- Yacc automatically assigns numbers for tokens, but it can be overridden by

`%token NUMBER 621`

- Yacc also recognizes single characters as tokens. Therefore, assigned token numbers should no overlap ASCII codes.
- The definition part can include C code external to the definition of the parser and variable declarations, within `%{and %}` in the first column.
- It can also include the specification of the starting symbol in the grammar:

`%start nonterminal`

- The rules part contains grammar definition in a modified BNF form.
- Actions is C code in `{ }` and can be embedded inside (Translation schemes).

Input File: Auxiliary Routines Part:

- The auxiliary routines part is only C code.
- It includes function definitions for every function needed in rules part.
- It can also contain the `main()` function definition if the parser is going to be run as a program.
- The `main()` function must call the function `yyparse()`.

Input File:

- If `yylex()` is not defined in the auxiliary routines sections, then it should be included:
`#include "lex.yy.c"`

- YACC input file generally finishes with:

`.y`

Output Files:

- The output of YACC is a file named **y.tab.c**
- If it contains the **main()** definition, it must be compiled to be executable.
- Otherwise, the code can be an external function definition for the function **int yyparse()**
- If called with the **-d** option in the command line, Yacc produces as output a header file **y.tab.h** with all its specific definition (particularly important are token definitions to be included, for example, in a Lex input file).
- If called with the **-v** option, Yacc produces as output a file **y.output** containing a textual description of the LALR(1) parsing table used by the parser. This is useful for tracking down how the parser solves conflicts.

Semantics

Syntax Directed Translation:

- A formalist called as syntax directed definition is used for specifying translations for programming language constructs.
- A syntax directed definition is a generalization of a context free grammar in which each grammar symbol has associated set of attributes and each and each productions is associated with a set of semantic rules

Definition of (syntax Directed definition) SDD :

- SDD is a generalization of CFG in which each grammar productions $X \rightarrow \alpha$ is associated with it a set of semantic rules of the form

$a: = f(b_1, b_2, \dots, b_k)$

Where a is an attributes obtained from the function f .

A syntax-directed definition is a generalization of a context-free grammar in which:

- Each grammar symbol is associated with a set of attributes.
- This set of attributes for a grammar symbol is partitioned into two subsets called synthesized and inherited attributes of that grammar symbol.
- Each production rule is associated with a set of semantic rules.
- Semantic rules set up dependencies between attributes which can be represented by a dependency graph.
- This dependency graph determines the evaluation order of these semantic rules.
- Evaluation of a semantic rule defines the value of an attribute. But a semantic rule may also have some side effects such as printing a value.

The two attributes for non terminal are :

The two attributes for non terminal are :

Synthesized attribute (S-attribute) : (\uparrow)

An attribute is said to be synthesized attribute if its value at a parse tree node is determined from attribute values at the children of the node

Inherited attribute: (\uparrow, \rightarrow)

An inherited attribute is one whose value at parse tree node is determined in terms of attributes at the parent and | or siblings of that node.

- The attribute can be string, a number, a type, a, memory location or anything else.
- The parse tree showing the value of attributes at each node is called an annotated parse tree.

The process of computing the attribute values at the node is called annotating or decorating the parse tree. Terminals can have synthesized attributes, but not inherited attributes.

Annotated Parse Tree

- A parse tree showing the values of attributes at each node is called an Annotated parse tree.
- The process of computing the attributes values at the nodes is called annotating (or decorating) of the parse tree.
- Of course, the order of these computations depends on the dependency graph induced by the semantic rules.

Ex1:1) Synthesized Attributes : Ex: Consider the CFG :

$S \rightarrow EN$

$E \rightarrow E+T$

$E \rightarrow E-T$

$E \rightarrow T$

$T \rightarrow T * F$

$T \rightarrow T / F$

$T \rightarrow F$

$F \rightarrow (E)$

$F \rightarrow \text{digit}$ $N \rightarrow ;$

Solution: The syntax directed definition can be written for the above grammar by using semantic actions for each production

Productionrule

$S \rightarrow EN$
 $E \rightarrow E1 + T$
 $E \rightarrow E1 - T$
 $E \rightarrow T$
 $T \rightarrow T * F$
 $T \rightarrow T | F$
 $F \rightarrow (E)$
 $T \rightarrow F$
 $F \rightarrow \text{digit}$
 $N \rightarrow ;$

Semanticactions

$S.val = E.val$
 $E.val = E1.val + T.val$
 $E.val = E1.val - T.val$
 $E.val = T.val$
 $T.val = T.val * F.val$
 $T.val = T.val | F.val$
 $F.val = E.val$
 $T.val = F.val$
 $F.val = \text{digit.lexval}$
 can be ignored by lexical Analyzer as; I is terminating symbol

For the Non-terminals E, T and F the values can be obtained using the attribute "Val".

The taken digit has synthesized attribute "lexval".

In $S \rightarrow EN$, symbol S is the start symbol. This rule is to print the final answer of expressed.

Following steps are followed to Compute S attributed definition

Write the SDD using the appropriate semantic actions for corresponding production rule of the given Grammar.

The annotated parse tree is generated and attribute values are computed. The Computation is done in bottom up manner.

The value obtained at the node is supposed to be final output.

L-attributed SDT

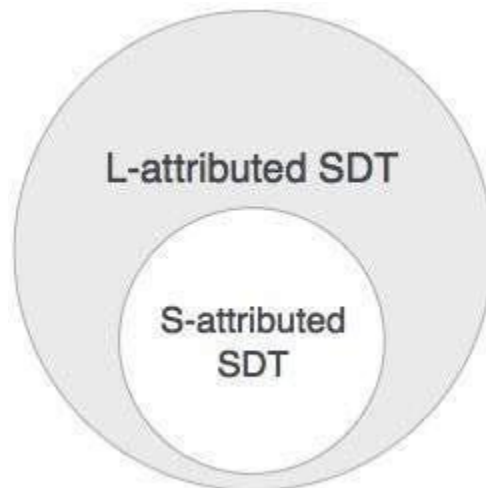
This form of SDT uses both synthesized and inherited attributes with restriction of not taking values from right siblings.

In L-attributed SDTs, a non-terminal can get values from its parent, child, and sibling nodes. As in the following production

$S \rightarrow ABC$

S can take values from A, B, and C (synthesized). A can take values from S only. B can take values from S and A. C can get values from S, A, and B. No non-terminal can get values from the sibling to its right.

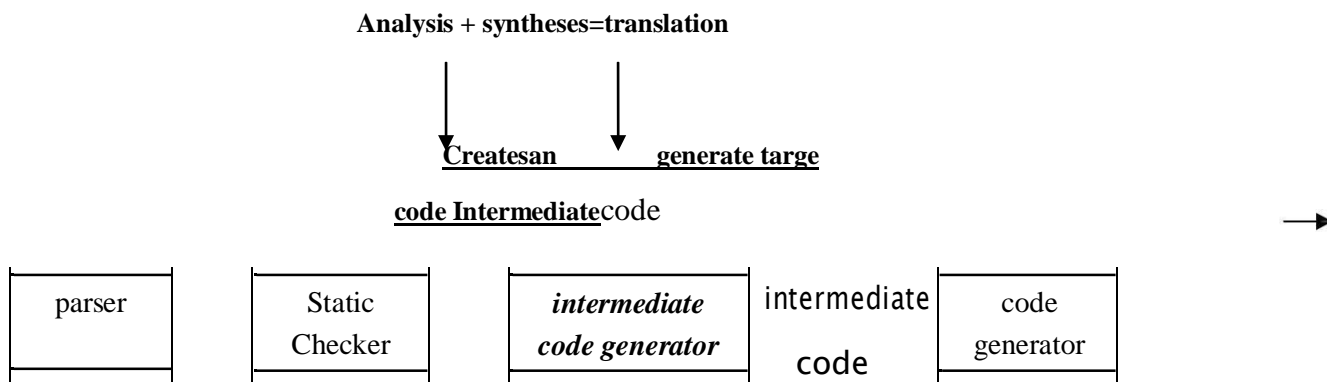
Attributes in L-attributed SDTs are evaluated by depth-first and left-to-right parsing manner.



We may conclude that if a definition is S-attributed, then it is also L-attributed as L-attributed definition encloses S-attributed definitions

Intermediate Code

An intermediate code form of source program is an internal form of a program created by the compiler while translating the program from a high-level language to assembly code(or)object code(machine code).an intermediate source form represents a more attractive form of target code than does assembly. An optimizing Compiler performs optimizations on the intermediate source form and produces an objectmodule.



In the analysis –synthesis model of a compiler, the front-end translates a source program into an intermediate representation from which the back-end generates target code, in many compilers the source code is translated into a language which is intermediate in complexity between a HLL and machine code .the usual intermediate code introduces symbols to stand for various temporary quantities.

We assume that the source program has already been parsed and statically checked..the various intermediate code forms are:

- a) Polishnotation
- b) Abstract syntax trees(or)syntaxtrees
- c) Quadruples
- d) Triples three address code
- e) Indirecttriples
- f) Abstract machinecode(or)pseudocopde

postfix notation:

The ordinary (infix) way of writing the sum of a and b is with the operator in the middle: $a+b$. the postfix (or postfix polish) notation for the same expression places the operator at the right end, $asab+$.

In general, if e_1 and e_2 are any postfix expressions, and \emptyset to the values denoted by e_1 and e_2 is indicated in postfix notation by $e_1e_2\emptyset$.no parentheses are needed in postfix notation because the position and priority (number of arguments) of the operators permits only one way to decode a postfix expression.

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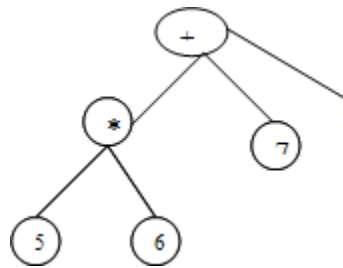
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Syntax tree:



Annotated parse tree :

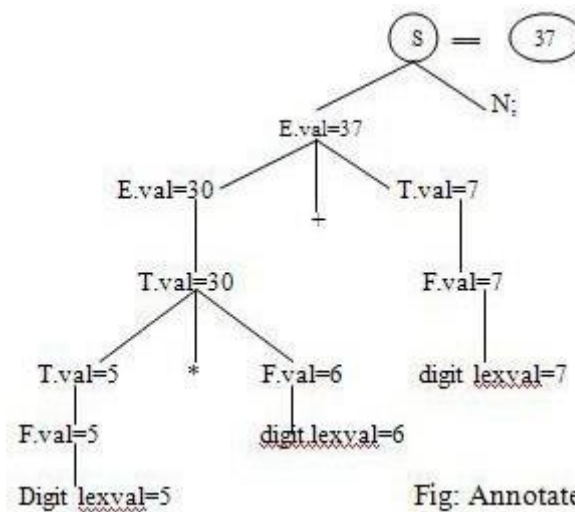


Fig: Annotated parse tree

ASSIGNMENT STATEMENTS

Suppose that the context in which an assignment appears is given by the following grammar. P

□ M D

M □ ε

D □ D ; D | **id** : T | **proc id** ; N D ; S

N □ ε

Nonterminal P becomes the new start symbol when these productions are added to those in the translation scheme shown below.

Translation scheme to produce three-address code for assignments

$S \rightarrow id := E$ { p := lookup (id.name);
 if p ≠ nil **then**
 emit(p ' := ' E.place)
 else error }

$E \rightarrow E_1 + E_2$ { E.place := newtemp;

```

                                emit(E.place ': =' E1.place ' + ' E2.place ) }
E → E1 * E2    { E.place := newtemp;
                                emit(E.place ': =' E1.place ' * ' E2.place ) }
E → - E1        { E.place := newtemp;
                                emit ( E.place ': =' 'uminus' E1.place ) }
E → ( E1 )      { E.place := E1.place }

E → id          { p := lookup ( id.name);
                                if p ≠ nil then
                                    E.place := p
                                else error }

```

Flow-of-Control Statements

We now consider the translation of boolean expressions into three-address code in the context of if-then, if-then-else, and while-do statements such as those generated by the following grammar:

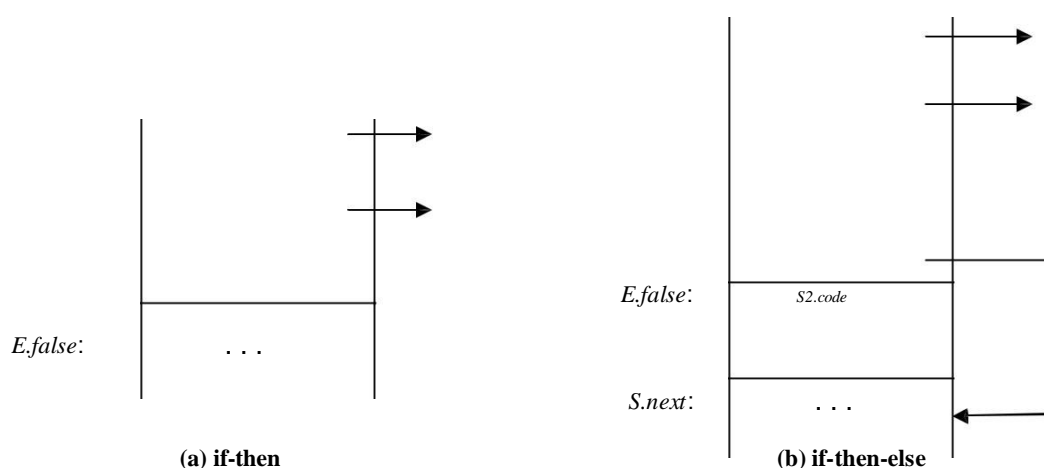
```

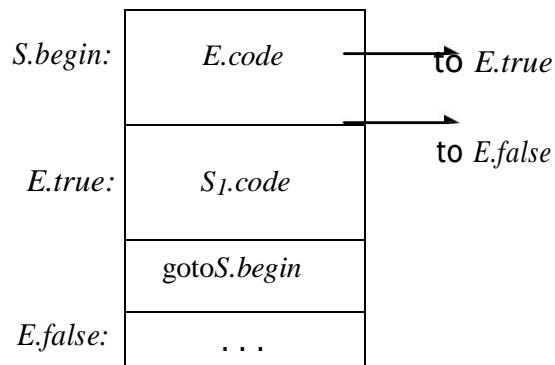
S → if E then S1
    if E then S1 else
        | S2
    | while E do S1

```

In each of these productions, E is the Boolean expression to be translated. In the translation, we assume that a three-address statement can be symbolically labeled, and that the function *newlabel* returns a new symbolic label each time it is called.

- $E.true$ is the label to which control flows if E is true, and $E.false$ is the label to which control flows if E is false.
- The semantic rules for translating a flow-of-control statement S allow control to flow from the translation $S.code$ to the three-address instruction immediately following $S.code$.
- $S.next$ is a label that is attached to the first three-address instruction to be executed after the code for **Code for if-then , if-then-else, and while-do statements**





(c) while-do

PRODUCTION	SEMANTIC RULES
$S \xrightarrow{\quad} \text{if } E \text{ then } S_1$	$E.true := \text{newlabel};$ $E.false := S.next;$ $S_1.next := S.next;$ $S.code := E.code \parallel \text{gen}(E.true, ",:") \parallel S_1.code$
$S \xrightarrow{\quad} \text{if } E \text{ then } S_1 \text{ else } S_2$	$E.true := \text{newlabel};$ $E.false := \text{newlabel};$ $S_1.next := S.next;$ $S_2.next := S.next;$ $S.code := E.code \parallel \text{gen}(E.true, ",:") \parallel S_1.code \parallel$ $\text{gen}(, \text{goto } S.next) \parallel$ $\text{gen}(E.false, ",:") \parallel S_2.code$
$S \xrightarrow{\quad} \text{while } E \text{ do } S_1$	$S.begin := \text{newlabel};$ $E.true := \text{newlabel};$ $E.false := S.next;$ $S_1.next := S.begin;$ $S.code := \text{gen}(S.begin, ",:") \parallel E.code \parallel$ $\text{gen}(E.true, ",:") \parallel S_1.code \parallel$ $\text{gen}(, \text{goto } S.begin)$